

Unified Flavor Symmetry from warped dimensions

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We propose a scenario which accommodates all the masses and mixings of the SM fermions in a model of warped extra-dimensions with all matter fields in the bulk. In this scenario, the same flavor symmetric structure is imposed on all the fermions of the Standard Model (SM), including neutrinos. Due to the exponential sensitivity on bulk fermion masses, a small breaking of the symmetry can be greatly enhanced and produce seemingly un-symmetric hierarchical masses and small mixing angles among the charged fermion zero-modes (SM quarks and charged leptons) and wash-out the obvious effects of the symmetry. With the Higgs field leaking into the bulk, and Dirac neutrinos sufficiently localized towards the UV boundary, the neutrino mass hierarchy and flavor structure will still be largely dominated by the fundamental flavor structure. The neutrino sector would then reflect the fundamental flavor structure, whereas the quark sector would probe the effects of the flavor symmetry breaking sector. As an example, we explore these features in the context of a family permutation symmetry imposed in both quark and lepton sectors.

The original motivation for warped extra-dimensions, or Randall-Sundrum models (RS), was to address the hierarchy problem. In RS the fundamental scale of gravity is exponentially reduced from the Planck mass scale to a TeV size due to a Higgs sector localized near the boundary of the extra dimension [1]. If SM fermions are allowed to propagate in the extra dimension [2], and become localized towards either boundary, the scenario also addresses the flavor problem of the SM and suppresses generic flavor-violating higher-order operators present in the original RS setup. However, KK-mediated processes still generate dangerous contributions to electroweak and flavor observables [3–5], pushing the KK scale to 5 – 10 TeV [6]. One realization of the model is based on the so-called flavor anarchy [5], in which one assumes that no special structure governs the flavor of Yukawa couplings and bulk fermion masses, as natural $\mathcal{O}(1)$ values for these 5D parameters already generate viable masses and mixings. The neutrino sector must behave differently, first due to the possibility of Majorana mass terms, and second because this setup generates large mass hierarchies and small mixing angles, at odds with neutrino observations. An interesting property of warped scenarios was investigated in [7], for the case of a bulk Higgs wave function leaking into the extra dimension. There one would obtain small mixing angles and hierarchical masses for all charged fermions, and at the same time very small Dirac masses, with large mixing angles and negligible mass hierarchy for neutrinos. Thus the flavor anarchy paradigm could still work in these scenarios.

Here we present a scenario where instead of adopting flavor anarchy, we propose that all fermions share the

same flavor symmetry. We assume that the flavor violating effects in the 5D Lagrangian can be parametrized by a small coefficient whose size is controlled by a ratio of scales, $\frac{\langle \phi \rangle^n}{\Lambda^n}$, with $\langle \phi \rangle$ the vacuum expectation value (VEV) of some flavon field, and Λ some cut-off mass scale, or the KK mass of some other flavon fields. This small breaking of the flavor symmetry is enough to reproduce correctly the flavor structure of the SM in both the quark and lepton sectors. The (stable) static spacetime background is:

$$ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (1)$$

where the extra coordinate y ranges between the two boundaries at $y = 0$ and $y = y_{\text{TeV}}$, and where $A(y)$ is the warp factor responsible for exponentially suppressing mass scales at different slices of the extra dimension. In the original RS scenario $A(y) = ky$, with k the curvature scale of the AdS_5 interval, while in general warped scenarios $A(y)$ is a more general (growing) function of y . The appeal of more complicated metrics lies on the possibility of having light KK resonances (~ 1 TeV), while keeping flavor and precision electroweak bounds at bay [8, 9]. For simplicity, we assume $A(y) = ky$, unless otherwise specified. Assuming invariance under the usual SM gauge group, the 5D quark Lagrangian is

$$\begin{aligned} \mathcal{L}_q = & \mathcal{L}_{\text{kinetic}} + M_{q_i} \bar{Q}_i Q_i + M_{u_i} \bar{U}_i U_i + M_{d_i} \bar{D}_i D_i \\ & + (Y_{ij_u}^{5D} H \bar{Q}_i U_j + h.c.) + (Y_{ij_d}^{5D} H \bar{Q}_i D_j + h.c.), \end{aligned} \quad (2)$$

where Q_i , U_i and D_i are 5D quarks (doublets and singlets under $SU(2)$). In the lepton sector, we assume that Majorana mass terms are forbidden, and so the Lagrangian

can be trivially obtained from the previous one by substituting Q_i by L_i , U_i by N_i and D_i by E_i , where L_i are lepton doublets, and N_i and E_i are neutrino and lepton singlets, respectively. The Higgs field H is a bulk scalar that acquires a nontrivial VEV $v(y) = v_0 e^{aky}$, exponentially localized towards the TeV boundary, with delocalization controlled by the parameter a . Such nontrivial exponential VEV-s appear naturally in warped backgrounds with simple scalar potentials and appropriate boundary conditions. This extra dimensional scenario has two sources of flavor. One is the usual Yukawa couplings Y_{ij}^u , Y_{ij}^d , Y_{ij}^e and Y_{ij}^ν (dimensionless parameters defined in units of the curvature out the dimension-full 5D Yukawa couplings as $Y_{ij}^{5D} = \sqrt{k} Y_{ij}$). The other comes from the fermion bulk mass terms, diagonal in flavor space, taken to be constant bulk terms written in units of the curvature k , i.e. $M_i = c_i k$ ($M_i = M_{q_i}, M_{u_i}, M_{d_i}, M_{L_i}, M_{\nu_i}, M_{e_i}$).

As noted in [7], whenever the bulk Higgs localization parameter a is small enough in comparison with the c_i parameters, (i.e., for the Higgs sufficiently delocalized from the TeV brane), the 4D effective masses depend exponentially on a rather than on the c_i parameters. The effective 4D masses for all the SM fermions become

$$m_t = v \tilde{Y}_{33} \quad c_{q3}, c_{u3} < 1/2 \quad (3)$$

$$(m_f)_{ij} = v \epsilon^{(c_{L_i} - \frac{1}{2})} \epsilon^{(c_{R_j} - \frac{1}{2})} \tilde{Y}_{ij} \quad a > c_{L_i} + c_{R_j} \quad (4)$$

$$(m_\nu)_{ij} = v \epsilon^{a-1} \tilde{Y}_{ij} \quad a < c_{l_i} + c_{\nu_j}, \quad (5)$$

where m_t is the top quark mass, $(m_f)_{ij}$ are mass matrices for light quarks and charged leptons, and $(m_\nu)_{ij}$ is the Dirac neutrino mass matrix. The parameters $c_{L_i} \equiv c_{q_i}, c_{l_i}$ correspond to the $SU(2)$ doublets, and $c_{R_j} \equiv c_{u_j}, c_{d_j}, c_{e_j}, c_{\nu_j}$ are for the $SU(2)$ singlets. The warp factor ϵ defined by the background parameters as $\epsilon = e^{-ky_{TeV}} \sim 10^{-15}$ encapsulates the hierarchy between the UV (gravity) brane and the TeV (SM) brane. The c -parameters dependence of masses is shown in Fig. 1 for the case of a brane localized Higgs VEV ($a = 30$) and for a delocalized Higgs VEV ($a = 1.9$), where the appearance of a plateau in the neutrino mass region reflects the insensitivity to the c_i values in that limit. A source of tension arises since, in order to generate viable neutrino masses from equation (5), one requires that $a \sim 1.80 - 1.95$. Values of $a < 2$ will reintroduce some amount of tuning in the model and, for example, for $a = 1.95$, some independent parameters of the 5D Higgs potential must be fixed to be equal to within about 1%. However, this same tuning will also be responsible for generating a light enough Higgs mode compared to the KK scale [8]. In more general warped backgrounds this tension can easily disappear due to an enlarged parameter space, justifying the choice $a = 1.9$ throughout the rest of the paper.

We assume that all Yukawa matrices and fermion bulk masses from the 5D Lagrangian share the same symmetry

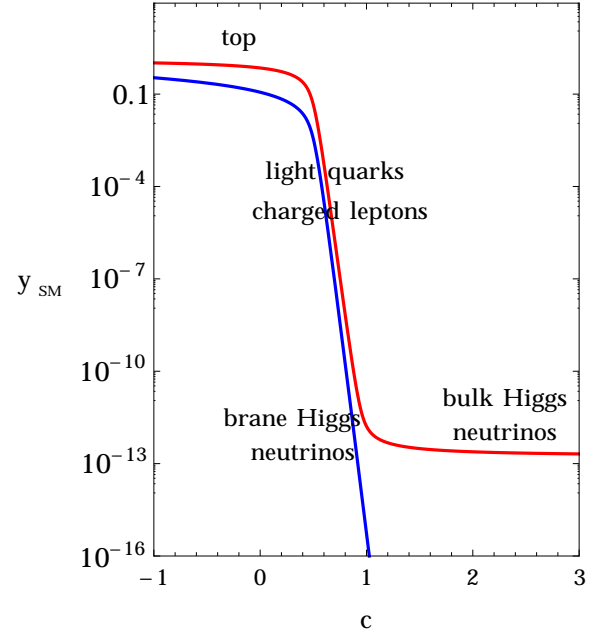


FIG. 1. Effective 4D Yukawa couplings for fermions as a function of the fermion bulk mass parameter c . For simplicity, we have taken the c -parameters for the doublet and the singlet to be equal.

structure broken by small terms i.e.

$$\mathbf{c}_f = \mathbf{c}_f^0 + \delta \mathbf{c}_f \quad (6)$$

$$Y_Y = Y_Y^0 + \delta Y_Y, \quad (7)$$

for all fermions of the model with $Y_Y = Y_u, Y_d, Y_e, Y_\nu$ and $\mathbf{c}_f = \mathbf{c}_q, \mathbf{c}_u, \mathbf{c}_d, \mathbf{c}_l, \mathbf{c}_e, \mathbf{c}_\nu$. The matrices Y_Y^0 and \mathbf{c}_f^0 are flavor symmetric, while the small corrections $\delta \mathbf{c}_f$ and δY_Y do not have *a priori* any flavor structure.

From Eqs. (3)–(5), the fermion masses receive different corrections due to flavor violating terms:

$$m_t = m_t^0 + \delta m_t \quad c_{q3}, c_{u3} < 1/2 \quad (8)$$

$$(m_f)_{ij} = (m_f)_{ij}^0 \epsilon^{(\delta c_{L_i} + \delta c_{R_j})} \quad a > c_{L_i} + c_{R_j} \quad (9)$$

$$(m_\nu)_{ij} = (m_\nu)_{ij}^0 + \delta (m_\nu)_{ij} \quad a < c_{l_i} + c_{\nu_j}. \quad (10)$$

The exponential sensitivity on the c -parameters is responsible for an exponential sensitivity of symmetry breaking terms. Since $\epsilon \sim 10^{-15}$, the corrections to the mass matrices caused by these are of order $\sim 10^{-15(\delta c_i + \delta c_j)}$, which means they could account for the observed hierarchies in the quark and charged lepton masses, as long as the symmetry breaking corrections δc_i remain between -0.1 and $+0.1$. The mixing angles are also exponentially sensitive to small symmetry breaking terms so that the mixing angles diagonalizing the mass matrices from the left will be $V_{ij} \sim \epsilon^{(\delta c_{L_i} - \delta c_{L_j})}$ for $(i < j)$ for quarks and charged leptons. Thus effects of the original symmetry are washed out in the quark and charged lepton sectors, while in the Dirac neutrino sector the sensitivity to the symmetry breaking is linear (i.e. small).

To qualify our assertions, we impose a simple structure for all the flavor parameters of the model, namely one which remains invariant under family permutations [10]. This leads to a flavor structure where the 5D Yukawa couplings are invariant under $S_3 \times S_3$, while the 5D fermion bulk mass matrices are invariant under S_3 . This leads to democratic 5D Yukawa couplings and to 5D fermion bulk mass matrices parametrized as

$$Y_Y^D \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{c}_f^D = \begin{pmatrix} A_f & B_f & B_f \\ B_f & A_f & B_f \\ B_f & B_f & A_f \end{pmatrix}. \quad (11)$$

Since all flavor structure is described by Eq. (11), we simultaneously diagonalize all matrices and obtain

$$Y_Y^0 = y_Y^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{c}_f^0 = \begin{pmatrix} c_{f1}^0 & 0 & 0 \\ 0 & c_{f1}^0 & 0 \\ 0 & 0 & c_{f3}^0 \end{pmatrix}, \quad (12)$$

where $y_Y^0 = y_u, y_d, y_e, y_\nu$ are complex Yukawa couplings in the up, down, charged lepton and neutrino Yukawa sectors. The matrix \mathbf{c}_f is in its diagonal basis with real entries and \mathbf{c}_f^0 . Democratic mass matrices produce two massless fermions and one massive one. The 0-th order CKM and PMNS matrices can be parametrized as

$$V_i^0 = \begin{pmatrix} \cos \theta_i^0 & \sin \theta_i^0 & 0 \\ -\sin \theta_i^0 & \cos \theta_i^0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (13)$$

where $i = \text{CKM, PMNS}$. Both matrices contain an angle θ_i^0 , not fixed by the $S_3 \times S_3$ symmetry, but by the symmetry breaking terms, implemented in Eqs. (6) and (7). The small breaking of the symmetry is responsible for a small lift of the zero masses, yielding a viable neutrino spectrum, with one heavier and two lighter eigenstates with similar masses, and a *normal* hierarchy ordering:

$$m_3 \sim \frac{m_\nu^0}{\sqrt{|\epsilon^{(2c_{l3}-1)} - 1|}}, \quad (14)$$

$$m_1 \sim \delta Y_\nu m_\nu^0 \quad m_2 \sim \delta Y_\nu m_\nu^0, \quad (15)$$

where $m_\nu^0 = v \epsilon^{a-1}$. We have kept the term in c_{l3} since it can have an effect when $c_{l3} \lesssim \frac{1}{2}$. Neutrino mass data require $m_\nu^0 \sim (0.05 - 0.1)$ eV, which in turn fixes the size of the Higgs localization parameter a . The dependence on δY_ν is evident while the δc_i 's are basically free (even the 0-th order $c_{\nu i}^0$ are relatively free, as long as they satisfy $a < c_\nu + c_l$). For example, one finds that $\delta Y_\nu \lesssim \sqrt{r} \sim 0.17$, to generate a viable neutrino mass hierarchy ratio $r = (|m_2|^2 - |m_1|^2)/(|m_3|^2 - |m_1|^2) \sim 0.03$.

In the charged lepton, up-and down-quark sectors, the massless states are also lifted by the flavor symmetry breaking leaving a suppression proportional to δY . In addition, the exponential dependence on the symmetry breaking parameters δc_{fi} creates a hierarchy among all

the masses. The third generation charged fermion masses are

$$m_t \sim m_t^0, \quad (16)$$

$$m_b \sim m_b^0 \epsilon^{\delta c_{e3}}, \quad (17)$$

$$m_\tau \sim m_\tau^0 \epsilon^{(\delta c_{l3} + \delta c_{e3})}, \quad (18)$$

with the 0-th order masses¹ $m_t^0 = y_u^0 v$, $m_b^0 = y_d^0 v \epsilon^{c_{d3}-1/2}$ and $m_\tau^0 = y_e^0 v \epsilon^{(c_{l3}+c_{e3}-1)}$. The lighter fermion masses are

$$m_{f2} \sim \delta Y_Y m_{f1}^0 \epsilon^{(\delta c_{L2} + \delta c_{R2})}, \quad (19)$$

$$m_{f1} \sim \delta Y_Y m_{f1}^0 \epsilon^{(\delta c_{L1} + \delta c_{R1})}, \quad (20)$$

where $m_{f2} \equiv m_c, m_s, m_\mu$, $m_{f1} \equiv m_u, m_d, m_e$ and $m_{f1}^0 = v \epsilon^{(c_{L1}+c_{R1}-1)}$. Note that in the flavor symmetric limit, the electron and muon, the down and strange quarks, and the up and charm quarks, are massless. The symmetry breaking produces non-zero masses proportional to the generic size of δY among these fermions², with an added source of hierarchy due to the exponential dependence on the δc_i . It is quite simple in this scenario to obtain phenomenologically viable quark and lepton masses by appropriately fixing the different δc_i within the constraint $|\delta c_i| \lesssim 0.1$. The hierarchies between fermion masses occur naturally and are under control since they depend exponentially on *small* numbers (they are hierarchical but not too hierarchical). Some masses and mixings still depend linearly on δY so that the typical size of these terms cannot be too small since, for instance, $\delta Y \gtrsim m_t/m_c$ in the (extreme) limit where the charm quark c-parameters are top-like.

The observed mixing angles in the CKM and PMNS matrices can also be generated in this unified scenario. The CKM entries become³

$$V_{us} \sim \epsilon^{(\delta c_{q1} - \delta c_{q2})}, \quad (21)$$

$$V_{cb} \sim \delta Y \epsilon^{(\delta c_{q2} - \delta c_{q3})} \quad (22)$$

$$V_{ub} \sim \delta Y \epsilon^{(\delta c_{q1} - \delta c_{q3})}. \quad (23)$$

With respect to the 0-th order CKM matrix from Eq. (13), V_{us} receives a suppression exponentially sensitive to the difference between two small terms with respect to the CKM, which can easily reproduce the

¹ Assuming that $c_{q3}^0, c_{u3} < \frac{1}{2}$ and $c_{d3}^0, c_{e3}^0, c_{l3}^0 > \frac{1}{2}$.

² The value of the light quark wavefunctions on the TeV brane is slightly higher than in usual RS scenarios to overcome the δY suppression. This induces stronger couplings with KK gluons and thus generically enhance dangerous FCNC processes. These dangerous effects can be addressed by invoking additional flavor constraints such as enforcing exactly $c_{d1} = c_{d2}$ [11], or going to more general warped backgrounds where flavor bounds can be much milder [8].

³ V_{cb} and V_{ub} receive an extra parametric suppression linear in the small Yukawa perturbations δY , caused by the broken S_3 symmetry.

Cabibbo angle. The angles V_{cb} and V_{ub} , lifted from the initial zero value, acquire a double suppression, one exponential, and one proportional to $\delta Y \sim 0.1$, with the ratio V_{cb}/V_{us} of the correct order of magnitude for the typical size for δY and δc , assuming an ordering $\delta c_{q1} > \delta c_{q2} > \delta c_{q3}$. The expected order of the ratio $V_{ub}/V_{cb} \sim V_{us}$ also remains realistic, up to order one factors not taken into account in the estimates. This last feature is generic in usual RS scenarios.

The parametric dependence of the PMNS entries is different:

$$V_{e2} \sim \sin \theta_\nu^0 \quad (24)$$

$$V_{e3} \sim \delta Y_{13}^\nu \sqrt{|\epsilon^{(2c_{l3}-1)} - 1|} \quad (25)$$

$$V_{\mu 3} \sim \delta Y_{23}^\nu \sqrt{|\epsilon^{(2c_{l3}-1)} - 1|}. \quad (26)$$

Contrary to the quark sector, the value of V_{e2} is not suppressed and remains generically of $\mathcal{O}(1)$, fixed by the structure of the neutrino Yukawa flavor violating matrix δY_{ij}^ν . The entries V_{e3} and $V_{\mu 3}$ are lifted from zero, both depend on δY and, not only are they not further suppressed by exponential terms, but can actually be enhanced by exponential terms (as long as the approximation remains valid). In particular if $c_{l3} \lesssim 1/2$, it is possible to lift the values of the mixing angles as shown in Eqs. (25) and (26). This feature is specific to the case $a < c_{l3} + c_{\nu_j}$ and $c_{l3} < 1/2$, and is not generic in usual RS scenarios. More precise (and less compact) formulae will be presented in a companion long paper.

The observed mixing angles in the neutrino sector are most sensitive to the flavor structure of the neutrino Yukawa matrix δY^ν , but not much to the charged lepton Yukawa matrix δY^l or to the δc_i . The bulk mass parameter of the third family lepton doublet should satisfy $c_{l3} < 1/2$ to easily obtain larger mixing angles for small $\delta Y \sim 0.1$ (given that $V_{\mu 3}^{\text{exp}} \sim 0.65$ and $V_{e3}^{\text{exp}} \sim 0.15$). This condition is very interesting as it is the same in the quark sector, where $c_{q3} \lesssim 1/2$, to obtain a large top quark mass, which could be a hint of an additional family symmetry among the $SU(2)$ doublets of the third family. Comparing expressions for the V_{PMNS} mixing angles and the neutrino masses, the element δY_{23} must be larger than the rest of δY so that $\delta Y \sim \delta Y_{13} \sim \frac{\delta Y_{23}}{4}$.

In this scenario it is easy to find a set of 0-th order bulk parameters that reproduce the SM and that show the features described above. For example, a working point for which the SM is a small perturbation (of order 10% around an S_3 symmetric set of parameters) away is shown in Table I. Charged fermions results are not too sensitive to small deviations in the Yukawa couplings ($\lesssim 0.1$), and once the δc_i 's are fixed, the δY 's can even be taken randomly as long as they remain at around 10%. One then obtains generically charged fermion masses and mixings consistent with the SM and any level of precision is possible by tuning these values.

f	q	u	d	l	ν	e
$c_{f1}^0 (\equiv c_{f2}^0)$	0.55	0.60	0.60	0.55	5.00	0.60
c_{f3}^0	0.40	0.40	0.50	0.40	2.00	0.60

TABLE I. Zeroth order 5D fermion c -parameters. For simplicity, we also set all the 0-th order Yukawa coefficients to be universal $y_u^0 = y_d^0 = y_\nu^0 = y_e^0 = 4.4$ and the Higgs localization parameter to $a = 1.9$.

In conclusion, we have proposed a general framework in warped extra dimensions where the SM flavor structure is unified in all fermion sectors. Small breaking terms are introduced for the 5D bulk mass and Yukawa parameters. The quark and charged lepton sectors are dominated by the small flavor breaking in the bulk c -parameters whereas the (Dirac) neutrino sector is dominated by flavor symmetry breaking Yukawa couplings. The main difference between these stems from allowing the Higgs field leak sufficiently out of the TeV brane so that the neutrino sector loses sensitivity on the 5D bulk mass parameters. A permutation symmetry was studied to illustrate the idea, but other symmetries can be invoked and explored within this framework, as will be further studied in a companion paper.

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- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); *ibid.*, Phys. Rev. Lett. **83**, 4690 (1999).
 - [2] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000); Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000).
 - [3] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308**, 050 (2003).
 - [4] G. Burdman, Phys. Rev. D **66**, 076003 (2002); G. Burdman, Phys. Lett. B **590**, 86 (2004); S. J. Huber, Nucl. Phys. B **666**, 269 (2003); M. S. Carena, A. Delgado, E. Ponton, T. M. P. Tait and C. E. M. Wagner, Phys. Rev. D **71**, 015010 (2005); *ibid.*, Phys. Rev. D **68**, 035010 (2003).
 - [5] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005).
 - [6] C. Csaki, A. Falkowski and A. Weiler, JHEP **0809**, 008 (2008); K. Agashe, A. Azatov and L. Zhu, Phys. Rev. D **79**, 056006 (2009).
 - [7] K. Agashe, T. Okui and R. Sundrum, Phys. Rev. Lett. **102**, 101801 (2009).
 - [8] J. A. Cabrer, G. von Gersdorff and M. Quiros, New J. Phys. **12**, 075012 (2010); *ibid.*, Phys. Lett. B **697**, 208 (2011); *ibid.*, JHEP **1105**, 083 (2011); *ibid.* Phys. Rev. D **84**, 035024 (2011); *ibid.*, JHEP **1201**, 033 (2012); M. Quiros, arXiv:1311.2824 [hep-ph].
 - [9] A. Carmona, E. Ponton and J. Santiago, JHEP **1110**, 137 (2011); S. Mert Aybat and J. Santiago, Phys. Rev. D **80**, 035005 (2009); A. Delgado and D. Diego, Phys. Rev. D **80**, 024030 (2009).
 - [10] Among the earliest works on flavor permutation symme-

- tries are: H. Harari, H. Haut and J. Weyers, Phys. Lett. B **78**, 459 (1978). S. Pakvasa and H. Sugawara, Phys. Lett. B**73**, 61 (1978); *ibid.*, Phys. Lett. B**82**, 105 (1979); E. Derman and H.S.Tsao, Phys. Rev. D **20**, 1207 (1979); Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. D **25**, 1895 (1982).
- [11] J. Santiago, JHEP **0812**, 046 (2008).